

# Quantum Gauge Freedom in Very Special Relativity

Sudhaker Upadhyay\*

*Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur-721302, West Bengal, India*

Prasanta K. Panigrahi†

*Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India*

We demonstrate Yokoyama gaugeon formalism for the Abelian one-form gauge (Maxwell) as well as for Abelian two-form gauge theory in the very special relativity (VSR) framework. In VSR scenario, the extended action due to introduction of gaugeon fields also possesses form invariance under quantum gauge transformations. It is observed that the gaugeon field together with gauge field naturally acquire mass, which is different from the conventional Higgs mechanism. The quantum gauge transformation implements a shift in gauge parameter. Further, we analyse the BRST symmetric gaugeon formalism in VSR which embeds only one subsidiary condition rather than two.

## I. OVERVIEW AND MOTIVATION

In recent times, the violations of Lorentz symmetry have been studied with great interest [1–6], though special relativity (SR), whose underlying Lorentz symmetry is valid at the largest energies available these days [7]. However, the violation of Lorentz symmetry has been considered as a possible evidence for Planck scale physics [8]. In this context, Cohen and Glashow [9] have proposed that the laws of physics need not be invariant under the full Lorentz group but rather under its subgroups that still preserves the basic elements of SR, like the constancy of the velocity of light. Any scheme whose space-time symmetries consist of translations along with any Lorentz subgroups is referred to as very special relativity (VSR). Most common subgroups fulfilling the essential requirements are the homothety group  $HOM(2)$  (with three parameters) and the similitude group  $SIM(2)$  (with four parameters) [9]. The generators of  $HOM(2)$  are  $T_1 = K_x + J_y$ ,  $T_2 = K_y - J_x$ , and  $K_z$ , where  $J$  and  $K$  are the generators of rotations and boosts, respectively. The generators of  $SIM(2)$  are  $T_1 = K_x + J_y$ ,  $T_2 = K_y - J_x$ ,  $K_z$  and  $J_z$ . These subgroups will be enlarged to the full Lorentz group when supplemented with discrete space-time symmetries  $CP$ . Recently, the three-dimensional supersymmetric Yang-Mills theory coupled to matter fields, (supersymmetric) Chern-Simons theory in  $SIM(1)$  superspace formalism [10] and  $SIM(2)$  superspace formalism [11] are derived. The Feynman rules and supergraphs [12] in  $SIM(2)$  superspace also has been studied.

VSR admits natural origin to lepton-number conserving neutrino masses without the need for sterile (right-handed) states [13]. This implies that neutrinoless double beta decay is forbidden, if VSR is solely responsible for neutrino masses. Further, VSR is generalized to  $N = 1$  SUSY gauge theories [14], where it is shown that these theories contain two conserved supercharges rather than the usual four. VSR is also modified by quantum corrections to produce a curved space-time with a cosmological constant [15], where it is shown that the symmetry group  $ISIM(2)$  does admit a 2-parameter family of continuous deformations, but none of these give rise to non-commutative translations analogous to those of the de-Sitter deformation of the Poincaré group. The VSR is generalized to curved space-times also, where it has been found that gauging the  $SIM(2)$  symmetry, which leaves the preferred null direction invariant, does not provide the complete couplings to the gravitational background [16]. The three subgroups relevant to VSR are also realized in the non-commutative space-time [17, 18] and in this setting the non-commutativity parameter  $\theta^{\mu\nu}$  behaves as lightlike. VSR has been generalized in various contexts.

---

\*Electronic address: sudhakerupadhyay@gmail.com

†Electronic address: pprasanta@iiserkol.ac.in; panigrahi.iiser@gmail.com

For example, the generalization of VSR ideas to de Sitter spacetime is studied where breaking of de Sitter invariance arises in two different ways [19]. This has also been shown that the event space underlying the dark matter and the dark gauge fields supports the algebraic structure underlying VSR [20]. A generalization of VSR in cosmology is also proposed where an anisotropic modification to the Friedmann-Robertson-Walker (FRW) line element occurs and for an arbitrarily oriented 1-form, the FRW space-time becomes of the Randers-Finsler type [21]. The VSR modifications to the quantum electrodynamics and the massive spin-1 particle are reported in Refs. [22, 23]. Furthermore, the generalization to the case of non-Abelian gauge fields is made in [24] and, in this context, the spontaneous symmetry-breaking mechanism to give a flavor-dependent VSR mass to the gauge bosons is also studied. VSR is also studied as background field theory, where averaging observable generates the nonlocal terms familiar from  $SIM(2)$  theories, while the short-distance behavior of the background field fermion propagator generates the infinite number of higher-order vertices of  $SIM(2)$ -quantum electrodynamics [25]. The electrostatic solutions as well as the VSR dispersion relations for Born-Infeld electrodynamics are investigated to be of a massive particle with nonlinear modifications in VSR scenario [26]. Recently, VSR generalization of the tensor field (reducible gauge) theories has also been analyzed using a Batalin-Vilkovisky (BV) formulation [27]. A rigorous construction of quantum field theory with a preferred direction is also studied very recently [28]. We would like to generalize the VSR in gaugeon formalism as gaugeon formalism is important in studying quantum gauge symmetry as well as in renormalization of gauge parameter.

The basic idea behind the gaugeon formalism [29] is to introduce the so-called gaugeon fields to the action which represent quantum gauge freedom. Originally, this formulation was developed in the case of quantum electrodynamics to settle the issues of renormalization of gauge parameter. In this connection the occurrence of shift in gauge parameter during renormalization [31] was addressed naturally by connecting theories in two different gauges within the same family by a  $q$ -number gauge transformation [29]. Further, this formalism has also been generalized to the case of Yang-Mills theory [30]. It has been found that gaugeon modes possess negative normed state which has been dealt with through Gupta-Bleuler type subsidiary condition. However, such condition is not applicable everywhere. For example, it fails when interaction is present between gaugeon fields. To improve the situation further, such subsidiary condition has been replaced by Kugo-Ojima type restrictions with the help of BRST charge [32–35]. The importance of BRST symmetry can be found in various contexts [36–40]. Though gaugeon formalism has been studied for various theories [32, 33, 41–48], it is still unexplored in the VSR context. We are interested to study the VSR effects on the gauge parameter, quantum gauge transformations and on the Yokoyama subsidiary conditions. This is the motivation of the present study.

Here, for illustration, we first consider Abelian 1-form (Maxwell) theory in VSR ( $SIM(2)$ -invariant) scenario and revisit the standard BRST quantization of the theory. Further to discuss the quantum gauge freedom, we introduce the gaugeon fields to the VSR action. Remarkably, terms containing gaugeon fields in the action remain local, even after breaking the full Lorentz invariance by some non-local terms. We found that the resulting action in VSR framework does not respect form invariance under the standard quantum gauge transformation. However, this action remains form-invariant under VSR-modified quantum gauge transformations. Also, the gauge parameter gets an automatic shift under this transformation even in VSR case. We also show that the gaugeon fields satisfy Proca equation, which signifies that these fields are massive. The introduction of gaugeon fields increases the physical degrees of freedom. To make it consistent with the original theory, we impose the Gupta-Bleuler type subsidiary condition, which removes the unphysical gaugeon modes from the theory. Further, to improve the situation with the Gupta-Bleuler type subsidiary, which has certain limitations, we demonstrate the BRST symmetric gaugeon formalism by introducing ghosts corresponding to gaugeon fields in VSR, which yields the more acceptable Kugo-Ojima subsidiary condition. This manifests the validity of gaugeon formalism of 1-form gauge theory in VSR. The novel observation here is that unlike to SR invariant case, the gaugeon fields together with the gauge fields get mass automatically. Furthermore, we explore Sakoda's technique in VSR for changing the gauge parameter of the linear covariant gauge from generating functional points of view with respect to the gauge freedom. We then generalize the obtained results for the case of Abelian 2-form gauge theory. For this purpose, we first consider Abelian 2-form gauge theory in VSR framework. The Abelian 2-form gauge theory, in a similar fashion to 1-form case, is also invariant under the modified gauge transformation. We compute the BRST symmetry of the theory. Moreover, to get consistent

description of quantum gauge freedom for 2-form gauge theory, we introduce gaugeon vector fields to the 2-form action. The resulting action remains invariant under VSR-modified quantum gauge symmetry. The gauge parameter translates under this symmetry for the 2-form case also, which is consistent with the renormalized gauge parameter. We observe that the vector gaugeon fields become massive in 2-form case also. We study the BRST symmetric gaugeon formalism for Abelian 2-form gauge theory in VSR. In this context, we show that the two subsidiary conditions to remove the unphysical fields are converted to a single but more general condition. Thus, the present investigation ensures the validity of gaugeon formalism in VSR.

The paper is organized in the following manner. In Sec. II, we derive both the gaugeon and BRST symmetric gaugeon formalism for Abelian 1-form gauge theory in VSR framework. In Sec. III, we generalize the discussion for both the gaugeon and BRST symmetric gaugeon formalism for Abelian 2-form gauge theory in VSR. We summarize our results in the last section and indicate directions for further study.

## II. 1-FORM GAUGE THEORY IN VSR

We analyse below the partially BRST symmetric and fully BRST symmetric gaugeon formalisms for 1-form gauge theory by extending the configuration space with the help of quantum fields. To make the theory consistent with the original one we remove the redundant degrees of freedom due to gaugeon fields with the help of suitable subsidiary condition.

### A. Brief review

In this subsection, we recapitulate the  $SIM(2)$ -invariant 1-form gauge theory in VSR [22, 24]. The most general gauge invariant action, quadratic in the gauge field, is given by

$$S = \int d^d x \left( -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}_\mu^\nu \right), \quad (1)$$

where  $n_\alpha$  is a chosen preferred null direction that transforms multiplicatively under a VSR transformation,  $g$  is a constant, and  $\tilde{F}_{\mu\nu}$  is wiggle field-strength tensor defined as

$$\tilde{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu A_\mu. \quad (2)$$

Here, the VSR mass parameter  $m$  sets the scale for the VSR effects and was introduced by a dimensional reason. The above action is not invariant under the standard gauge transformation. Rather, it is invariant under a VSR type gauge transformation:

$$\delta A_\mu = \partial_\mu \Lambda - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \Lambda, \quad (3)$$

where  $\Lambda$  is a local transformation parameter.

We handle the kind of non-local terms, present above, with the help of following definition [49]:

$$\frac{1}{n \cdot \partial} = \frac{1}{\partial_t + \partial_z} = \int_0^\infty da e^{-an \cdot \partial}. \quad (4)$$

From action (1) (with  $g = 0$ ), the equations of motion (EOM) for free Abelian field is calculated by,

$$\partial^\mu \tilde{F}_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \tilde{F}_{\mu\nu} = 0, \quad (5)$$

which further leads to

$$(\square - m^2)A_\nu = 0,$$

for a VSR type Lorentz gauge condition,  $\partial^\mu A_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu A_\mu = 0$ . This implies that the gauge field  $A_\mu$  has a mass  $m$  and the action (1) describes a massive gauge field.

To achieve the VSR-type Lorentz gauge in quantum action, we need to add the following term to the action (1):

$$S_{gf+gh}^L = \int d^d x \left\{ B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu + \frac{1}{2} \alpha B \right) + i c_\star (\square - m^2) c \right\},$$

where the last term is induced ghost terms with mass  $m$ . We note here that the gauge-fixed action in VSR-type axial gauge can also be constructed which has rather simpler form than the Lorenz gauge [27].

The effective action in VSR-type Lorentz gauge is given by,

$$S_{eff} = S + S_{gf+gh}^L, \quad (6)$$

which is not invariant under (VSR-type) gauge transformation but remains invariant under the following BRST transformation:

$$\begin{aligned} s_b A_\mu &= \partial_\mu c - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c, \\ s_b c &= 0, \\ s_b c_\star &= iB, \\ s_b B &= 0. \end{aligned} \quad (7)$$

The gauge-fixed action is BRST exact as it is evident from,

$$S_{gf+gh}^L = -i s_b \int d^d x \left\{ c_\star \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu + \frac{1}{2} \alpha B \right) \right\}.$$

We note that this gauge-fixed action, added to modify the classical action, has no contribution to the physical matrix elements of the theory. All the physical matrix elements of the theory are independent of the choice of the gauge-fixing parameter  $\alpha$ .

Next, we discuss the gaugeon formalism for 1-form gauge theory in VSR.

## B. Gaugeon formalism

Following the standard technique, we introduce the gaugeon field  $Y$  and its associated field  $Y_\star$ , obeying Bose-Einstein statistics, to the effective action (6). With such introduction, the Yokoyama effective action for the Abelian one-form gauge theory in VSR follows,

$$\begin{aligned} S_Y &= \int d^d x \left( -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}_\mu^\nu + \partial_\mu B A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B A^\mu \right. \\ &\quad \left. - Y_\star [\square - m^2] Y + \frac{\varepsilon}{2} (Y_\star + \alpha B)^2 + i c_\star [\square - m^2] c \right), \end{aligned} \quad (8)$$

where  $\alpha$  is the gauge-fixing parameter and  $\varepsilon(\pm)$  is the sign factor. Here, we note that VSR effect does not change the spin-statistics theorem for the fields. The gaugeon action (8) remains form invariant under following quantum gauge transformations:

$$\begin{aligned} \delta_q A_\mu &= \tau \left( \partial_\mu Y - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu Y \right), \\ \delta_q Y_\star &= -\tau B, \\ \delta_q B &= 0, \quad \delta_q Y = 0, \\ \delta_q c &= 0, \quad \delta_q c_\star = 0, \end{aligned} \quad (9)$$

with a shift in gauge parameter

$$\hat{\alpha} = \alpha + \tau. \quad (10)$$

The two subsidiary conditions to remove the unphysical modes are

$$\begin{aligned} Q|\text{phys}\rangle &= 0, \\ Y_\star^{(+)}|\text{phys}\rangle &= 0. \end{aligned} \quad (11)$$

The first condition is the usual one of the standard formalism, which confines the unphysical gauge modes by the quartet mechanism. The second condition removes the gaugeon modes. The decomposition in positive and negative frequency parts are valid because of the free equation:

$$(\square - m^2)Y_\star = 0. \quad (12)$$

If this does not hold, the positive frequency part becomes ambiguous and the second subsidiary condition contradicts with time evolution. Eq. (12) reflects that the gaugeon field gets mass along with gauge and ghost fields in VSR. The photon two-point function in VSR is calculated by

$$D_{\mu\nu} = \frac{1}{(k^2 + m^2)^2} \left[ (k^2 + m^2)g_{\mu\nu} + (\epsilon\alpha^2 - 1) \left( k_\mu k_\nu + \frac{1}{2}m^2 \left( \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k} \right) + \frac{1}{4}m^4 \frac{n_\mu n_\nu}{(n \cdot k)^2} \right) \right], \quad (13)$$

where  $g = 0$  is considered for simplicity. It can be seen that the ultra-violet behavior of above two-point function  $\langle A_\mu A_\nu \rangle$  is  $O(1/k^2)$ , which coincides the standard two-point function of SR invariant theory.

### C. BRST symmetric gaugeon formalism

In this subsection, we discuss the BRST symmetric gaugeon formalism in VSR. For this purpose, we first extend the effective Yokoyama Lagrangian density (8), by introducing two Faddeev-Popov ghosts  $K, K_\star$  corresponding to the gaugeon fields as follows:

$$\begin{aligned} S_{BY} = & \int d^d x \left( -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}_\mu^\nu + \partial_\mu B A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B A^\mu \right. \\ & \left. - Y_\star [\square - m^2] Y + \frac{\epsilon}{2} (Y_\star + \alpha B)^2 + i c_\star [\square - m^2] c + i K_\star [\square - m^2] K \right). \end{aligned} \quad (14)$$

The BRST symmetry for the above action is written by,

$$\begin{aligned} s_b A_\mu &= \partial_\mu c - \frac{m^2}{n \cdot \partial} n_\mu c, \\ s_b c_\star &= -iB, \quad s_b c = 0, \\ s_b B &= 0, \quad s_b Y = K, \\ s_b K_\star &= -iY_\star, \quad s_b K = 0, \\ s_b Y_\star &= 0. \end{aligned} \quad (15)$$

The BRST charge using Noether's theorem is calculated by

$$Q_B = \int d^{d-1}x (c \overleftrightarrow{\partial}_0 B + K \overleftrightarrow{\partial}_0 Y_\star), \quad (16)$$

where  $\tilde{\partial}_0 = \partial_0 - \frac{m^2}{n \cdot \partial} n_0$ .

With the help of this BRST charge, we can define the physical subspace of the total Hilbert space satisfying,

$$Q_B|\text{phys}\rangle = 0. \quad (17)$$

It is interesting to observe that, even after getting extended by introduction of ghosts corresponding to gaugeon fields, the action admits quantum gauge transformation given by,

$$\begin{aligned}\delta_q A_\mu &= \tau \left( \partial_\mu Y - \frac{m^2}{n \cdot \partial} n_\mu Y \right), \\ \delta_q Y_\star &= -\tau B, \quad \delta_q B = 0, \\ \delta_q Y &= 0, \quad \delta_q c = \tau K, \\ \delta_q K_\star &= -\tau c_\star, \quad \delta_q c_\star = 0, \quad \delta_q K = 0.\end{aligned}\tag{18}$$

These transformations only shift gauge parameter leaving the action form-invariant. We observe that the above quantum gauge transformation commutes with the BRST transformation. This ensures that the BRST charge (16) is invariant under the quantum gauge transformation:

$$\delta_q Q_B = 0.\tag{19}$$

Therefore, the physical subspace is invariant under the quantum gauge transformation, as the physical subspace can be constructed with the help of BRST charge. However, the BRST symmetric gaugeon formalism is more acceptable in the sense that, this situation does not occur in partially BRST symmetric theory described in subsection B.

#### D. Sakoda's extension of the gauge freedom of the vector field in VSR

In this subsection, as a first step, we consider the Maxwell gauge field and starting with the generating functional for 1-form in the Landau gauge, we extend the gauge freedom by applying the Harada-Tsutsui gauge recovery procedure in the VSR context. Let us start with the effective action for the Maxwell theory in VSR-type Landau gauge (Eq. (6) with  $\alpha = 0$ ) given by

$$S_{eff} = \int d^d x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\nu \tilde{F}^{\nu\mu} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}_\mu^\alpha + B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu \right) + i c_\star (\square - m^2) c \right] \tag{20}$$

Now, the generating functional corresponding to this action is expressed by

$$Z = \int \mathcal{D}A \mathcal{D}B \mathcal{D}c_\star \mathcal{D}c \, e^{i S_{eff}} = \int \mathcal{D}A \mathcal{D}B \, \mathcal{W}_0, \tag{21}$$

with

$$\mathcal{W}_0 = \Delta \, e^{i \int d^d x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}_\mu^\nu + B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu \right) \right]}, \tag{22}$$

where  $\Delta = \det(\square - m^2)$ . Since the action in (22) contains the gauge fixing part together with the classical part, therefore, the functional  $\mathcal{W}_0$  is not invariant under the VSR-type gauge transformation,

$$\begin{aligned}\delta A_\mu &= \partial_\mu \Lambda - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \Lambda, \\ \delta B &= 0.\end{aligned}\tag{23}$$

In order to study the VSR-type gauge invariance of this gauge-fixed functional (quantum action), we promote the function  $\Lambda(x)$  to a dynamical variable and define

$$\mathcal{W}'_0 = \Delta \, e^{i \int d^d x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}_\mu^\nu + B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu + (\square - m^2) \Lambda \right) \right]}. \tag{24}$$

This functional  $\mathcal{W}'_0$  is found invariant under the extended VSR-type gauge transformation,

$$\begin{aligned}\delta A_\mu &= \partial_\mu \theta - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \theta, \\ \delta \Lambda &= -\theta, \\ \delta B &= 0.\end{aligned}\tag{25}$$

Because of this (quantum) gauge symmetry, the generating functional for  $\mathcal{W}'_0$ , defined as

$$Z = \int \mathcal{D}A\mathcal{D}B\mathcal{D}\Lambda \mathcal{W}'_0, \quad (26)$$

is divergent. In order to remove the extra degree of freedom, we need to fix the VSR-type gauge for  $\Lambda$ . Here, we consider the following VSR-type gauge-fixing condition:

$$-(\square - m^2)\Lambda = C. \quad (27)$$

Utilizing Faddeev-Popov trick, we write the generating functional as

$$Z = \int \mathcal{D}A\mathcal{D}B\mathcal{D}\Lambda \mathcal{W}'_0 \Delta\delta(-\square\Lambda + m^2\Lambda - C), \quad (28)$$

as  $\Delta\delta(-\square\Lambda + m^2\Lambda - C) = 1$ . By writing the Fourier integral for the delta functional with respect to  $B_\Lambda$  and applying 't Hooft averaging with a Gaussian weight, the generating functional reduces to,

$$Z = \int \mathcal{D}A\mathcal{D}B\mathcal{D}\Lambda\mathcal{D}B_\Lambda \mathcal{W}_1, \quad (29)$$

where

$$\mathcal{W}_1 = \Delta\Delta e^{i \int d^d x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}^{\nu\mu} + B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu + (\square - m^2)\Lambda \right) - B_\Lambda (\square - m^2)\Lambda + \frac{a}{2} B_\Lambda^2 \right]}. \quad (30)$$

The two Faddeev-Popov determinants in VSR are calculated as

$$\Delta\Delta = \int \mathcal{D}c_\star \mathcal{D}c \mathcal{D}\eta_\star \mathcal{D}\eta e^{i \int d^d x [i c_\star (\square - m^2)c + i \eta_\star (\square - m^2)\eta]}. \quad (31)$$

Consequently, we obtain the following Sakoda effective action for the extension of the gauge freedom:

$$\begin{aligned} S_{Sakoda} = & \int d^d x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\nu \tilde{F}^{\nu\mu} + B \left( \partial_\mu A^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu A^\mu + (\square - m^2)\Lambda \right) \right. \\ & \left. - B_\Lambda (\square - m^2)\Lambda + \frac{a}{2} B_\Lambda^2 + i c_\star (\square - m^2)c + i \eta_\star (\square - m^2)\eta \right]. \end{aligned} \quad (32)$$

This action enjoys the following extended VSR-type BRST transformation:

$$\begin{aligned} s_b A_\mu &= \partial_\mu c - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c, \quad s_b \Lambda = -(c + \eta), \\ s_b c &= 0, \quad s_b \eta = 0, \quad s_b c_\star = i B_\Lambda, \\ s_b \eta_\star &= i(B_\Lambda - B), \quad s_b B_\Lambda = 0, \quad s_b B = 0. \end{aligned} \quad (33)$$

Sakoda states that the effective action for the extension of the gauge freedom must be equivalent to the action of the gaugeon formalism [51]. Remarkably, the Sakoda effective action (32) coincides with the gaugeon action (8) in VSR framework after redefining the fields as,

$$\Lambda = \alpha Y, \quad B_\Lambda = \frac{1}{\alpha} Y_\star + B, \quad \eta = K, \quad \eta_\star = K_\star. \quad (34)$$

Here  $\alpha$  is a numerical parameter satisfying  $a = \epsilon\alpha^2$ . Thus, we note that the field  $\Lambda(x)$  introduced as an extended gauge freedom plays the character of a gaugeon field in VSR.

### III. ABELIAN 2-FORM GAUGE THEORY IN VSR

We start with the field-strength tensor in VSR for Kalb-Ramond tensor field  $B_{\mu\nu}$  [50] in VSR involving a fixed null vector  $n_\mu$ :

$$\begin{aligned} F_{\mu\nu\rho} = & \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} + \frac{1}{2}m^2 \left[ n_\mu \frac{1}{(n \cdot \partial)^2} n^\alpha (\partial_\nu B_{\rho\alpha} + \partial_\rho B_{\nu\alpha}) \right. \\ & \left. + n_\nu \frac{1}{(n \cdot \partial)^2} n^\alpha (\partial_\rho B_{\mu\alpha} + \partial_\mu B_{\rho\alpha}) + n_\rho \frac{1}{(n \cdot \partial)^2} n^\alpha (\partial_\mu B_{\nu\alpha} + \partial_\nu B_{\mu\alpha}) \right]. \end{aligned} \quad (35)$$

As before, the null vector  $n^\mu$  transforms multiplicatively under a VSR transformation to ensure the invariance of non-local terms. This field-strength tensor is not invariant under the standard gauge transformation  $\delta B_{\mu\nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu$ , where  $\zeta_\mu(x)$  is a vector parameter. One can dualize a two-form to a pseudoscalar in the VSR also satisfying modified Bianchi identity. Rather, it remains invariant under the following modified (VSR-type) gauge transformation:

$$\begin{aligned} \delta B_{\mu\nu} = & \tilde{\partial}_\mu \zeta_\nu - \tilde{\partial}_\nu \zeta_\mu, \\ = & \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \zeta_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \zeta_\mu. \end{aligned} \quad (36)$$

As before, to have the usual mass dimension for wiggle operator,  $\tilde{\partial}_\mu = \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu$ , a constant  $m$  has to be introduced.

The gauge invariant action in VSR to describe the massive Kalb-Ramond tensor field is given by

$$S_0 = \frac{1}{12} \int d^d x \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho}, \quad (37)$$

where the wiggle field-strength tensor has the following form:

$$\begin{aligned} \tilde{F}_{\mu\nu\rho} = & \tilde{\partial}_\mu B_{\nu\rho} + \tilde{\partial}_\nu B_{\rho\mu} + \tilde{\partial}_\rho B_{\mu\nu}, \\ = & \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_{\nu\rho} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu B_{\rho\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\rho B_{\mu\nu}, \\ = & F_{\mu\nu\rho} - \frac{1}{2} m^2 \left( n_\mu \frac{1}{(n \cdot \partial)^2} n^\alpha F_{\nu\rho\alpha} + n_\nu \frac{1}{(n \cdot \partial)^2} n^\alpha F_{\rho\mu\alpha} + n_\rho \frac{1}{(n \cdot \partial)^2} n^\alpha F_{\mu\nu\alpha} \right). \end{aligned} \quad (38)$$

It is evident from the above relation that,  $\tilde{F}_{\mu\nu\rho}$  does not coincide with  $F_{\mu\nu\rho}$  given in (35).

The EOM for Kalb-Ramond field is calculated as,

$$\tilde{\partial}_\mu \tilde{F}^{\mu\nu\rho} = 0. \quad (39)$$

For the VSR-type Lorentz gauge  $\tilde{\partial}_\mu B^{\mu\nu} = 0$ , the EOM reduces to

$$[\square - m^2] B^{\nu\rho} = 0, \quad (40)$$

which remarkably implies that the field  $B_{\mu\nu}$  has mass  $m$ .

The VSR-type Lorentz gauge can be implemented in the classical action by adding suitable gauge fixing and ghost terms. The gauge fixing and ghost action for antisymmetric rank 2 tensor field in VSR-type



Lorentz gauge is given by

$$\begin{aligned}
S_{gf+gh} &= \int d^d x \left[ i\bar{\rho}_\nu \tilde{\partial}_\mu (\tilde{\partial}^\mu \rho^\nu - \tilde{\partial}^\nu \rho^\mu) - \bar{\sigma} \tilde{\partial}_\mu \tilde{\partial}^\mu \sigma + \beta_\nu (\tilde{\partial}_\mu B^{\mu\nu} + \lambda_1 \beta^\nu - \tilde{\partial}^\nu \varphi) \right. \\
&\quad \left. - i\bar{\chi} (\tilde{\partial}_\mu \rho^\mu + \lambda_2 \chi) - i\bar{\rho}^\mu \tilde{\partial}_\mu \chi \right], \\
&= \int d^d x \left[ i\bar{\rho}_\nu \left( \partial_\mu \partial^\mu \rho^\nu - \partial_\mu \partial^\nu \rho^\mu - m^2 \rho^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \partial \cdot \rho + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\nu n \cdot \rho \right. \right. \\
&\quad \left. - \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} n^\nu n \cdot \rho \right) - \bar{\sigma} (\partial_\mu \partial^\mu - m^2) \sigma + \beta_\nu \partial_\mu B^{\mu\nu} - \frac{1}{2} m^2 \beta_\nu \frac{1}{n \cdot \partial} n_\mu B^{\mu\nu} + \lambda_1 \beta_\nu \beta^\nu \\
&\quad - \beta_\nu \partial^\nu \varphi + \frac{1}{2} \frac{m^2}{n \cdot \partial} \beta_\nu n^\nu \varphi - i\bar{\chi} \partial_\mu \rho^\mu + \frac{i}{2} m^2 \bar{\chi} \frac{1}{n \cdot \partial} n_\mu \rho^\mu - i\lambda_2 \bar{\chi} \chi - i\bar{\rho}^\mu \partial_\mu \chi \\
&\quad \left. - \frac{i}{2} \frac{m^2}{n \cdot \partial} \bar{\rho}^\mu n_\mu \chi \right], \tag{41}
\end{aligned}$$

where  $\lambda_1$  and  $\lambda_2$  are gauge parameters. It is evident from the above expression that the ghost fields and ghost of ghost fields have mass  $m$  in VSR as well.

### A. Gaugeon formalism

In this subsection, we study the Yokoyama gaugeon formalism to analyse the quantum gauge freedom for the Abelian rank-2 tensor field theory. We start with the effective Lagrangian density for a  $d$ -dimensional theory in Landau gauge,

$$\begin{aligned}
S_Y &= \int d^d x \left[ \frac{1}{12} \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} - i\tilde{\partial}_\mu \bar{\rho}_\nu (\tilde{\partial}^\mu \rho^\nu - \tilde{\partial}^\nu \rho^\mu) + \tilde{\partial}_\mu \bar{\sigma} \tilde{\partial}^\mu \sigma + \beta_\nu (\tilde{\partial}_\mu B^{\mu\nu} - \tilde{\partial}^\nu \varphi) + \epsilon (Y_\nu^* + \alpha \beta_\nu)^2 \right. \\
&\quad \left. - (\tilde{\partial}_\mu Y_\nu^* - \tilde{\partial}_\nu Y_\mu^*) \tilde{\partial}^\mu Y^\nu - i\bar{\chi} \tilde{\partial}_\mu \rho^\mu - i\chi (\tilde{\partial}_\mu \bar{\rho}^\mu - \lambda_2 \bar{\chi}) \right], \\
&= \int d^d x \left[ \frac{1}{12} \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} - i\bar{\rho}_\nu \left( \partial_\mu \partial^\mu \rho^\nu - \partial_\mu \partial^\nu \rho^\mu - m^2 \rho^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \partial \cdot \rho + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\nu n \cdot \rho \right. \right. \\
&\quad \left. - \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} n^\nu n \cdot \rho \right) - \bar{\sigma} (\partial_\mu \partial^\mu - m^2) \sigma + \beta_\nu \partial_\mu B^{\mu\nu} - \frac{1}{2} m^2 \beta_\nu \frac{1}{n \cdot \partial} n_\mu B^{\mu\nu} + \epsilon (Y_\nu^* + \alpha \beta_\nu)^2 \\
&\quad - \beta_\nu \partial^\nu \varphi + \frac{1}{2} \frac{m^2}{n \cdot \partial} \beta_\nu n^\nu \varphi - (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*) \partial^\mu Y^\nu + \frac{1}{2} (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*) \frac{m^2}{n \cdot \partial} n^\mu Y^\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} (n_\mu Y_\nu^* - n_\nu Y_\mu^*) \partial^\mu Y^\nu - \frac{1}{4} \frac{m^2}{n \cdot \partial} (n_\mu Y_\nu^* - n_\nu Y_\mu^*) \frac{m^2}{n \cdot \partial} n^\mu Y^\nu - i\bar{\chi} \partial_\mu \rho^\mu + \frac{i}{2} m^2 \bar{\chi} \frac{1}{n \cdot \partial} n_\mu \rho^\mu \\
&\quad \left. - i\lambda_2 \bar{\chi} \chi - i\bar{\rho}^\mu \partial_\mu \chi - \frac{i}{2} \frac{m^2}{n \cdot \partial} \bar{\rho}^\mu n_\mu \chi \right], \tag{42}
\end{aligned}$$

where  $Y_\nu$  and  $Y_\nu^*$  are the gaugeon fields respectively.

The Lagrangian density (42) is invariant under the following BRST transformations:

$$\begin{aligned}
s_b B_{\mu\nu} &= (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \rho_\mu), \\
s_b \rho_\mu &= -i\partial_\mu \sigma + i\frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \sigma, \quad s_b \sigma = 0, \\
s_b \bar{\rho}_\mu &= i\beta_\mu, \quad s_b \beta_\mu = 0, \quad s_b \bar{\sigma} = \bar{\chi}, \quad s_b \bar{\chi} = 0, \\
s_b \varphi &= \chi, \quad s_b \chi = 0, \quad s_b Y = 0, \quad s_b Y_* = 0. \tag{43}
\end{aligned}$$

Now, we demonstrate the following quantum gauge transformation, under which the Lagrangian density (42) remains form-invariant:

$$\begin{aligned}\delta_q B_{\mu\nu} &= \tau \left( \partial_\mu Y_\nu - \partial_\nu Y_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu Y_\mu \right), \\ \delta_q Y_\nu^* &= -\tau \beta_\nu, \\ \delta_q \Omega &= 0, \quad \Omega = \rho_\mu, \sigma, \bar{\rho}_\mu, \beta_\mu, \bar{\sigma}, \bar{\chi}, \varphi, \chi, Y_\nu,\end{aligned}\tag{44}$$

where  $\tau$  is an infinitesimal transformation parameter. The form-invariance of the Lagrangian density (42) under the quantum gauge transformation (44) reflects the following shift in parameter:

$$\alpha \longrightarrow \hat{\alpha} = \alpha + \tau \alpha.\tag{45}$$

Further, according to Yokoyama, to remove the unphysical gauge and gaugeon modes of the theory and to define physical states, one imposes two subsidiary conditions (the Kugo-Ojima type and Gupta-Bleuler type) as,

$$\begin{aligned}Q_B |\text{phys}\rangle &= 0, \\ (Y_\nu^*)^{(+)} |\text{phys}\rangle &= 0,\end{aligned}\tag{46}$$

where  $Q_B$  is the BRST charge. The expression for BRST charge using Noether's theorem is given by

$$Q_B = \int d^{d-1}x \left[ -2\tilde{F}^{0\nu\rho}(\tilde{\partial}_0 \rho_\nu - \tilde{\partial}_\nu \rho_0) + \beta_\nu(\tilde{\partial}^0 \rho^\nu - \tilde{\partial}^\nu \rho^0) - \tilde{\partial}_\nu \sigma(\tilde{\partial}^0 \bar{\rho}^\nu - \tilde{\partial}^\nu \bar{\rho}^0) + \bar{\chi} \tilde{\partial}^0 \sigma - \chi B^0 \right].\tag{47}$$

The Kugo-Ojima type subsidiary condition removes the unphysical modes corresponding to gauge field from the total Fock space. The Gupta-Bleuler type condition is used to remove the unphysical gaugeon modes from the physical states. The second subsidiary condition is valid, when  $Y_\nu^*$  satisfies the following free equation:

$$(\partial_\mu \partial^\mu - m^2) Y_\nu^* = 0,\tag{48}$$

which we have derived using equations of motion. The free equation (48) guarantees the decomposition of  $Y_\nu^*$  into positive and negative frequency parts. Consequently, the subsidiary conditions (46) warrant the positivity of the semi-definite metric of our physical state-vector space:

$$\langle \text{phys} | \text{phys} \rangle \geq 0,\tag{49}$$

and hence, we have a desirable physical subspace on which our unitary physical  $S$ -matrix exists.

### B. BRST symmetric gaugeon formalism

In this subsection we discuss the BRST symmetric gaugeon formalism for Abelian 2-form gauge theory, with the Lagrangian density:

$$\begin{aligned}
S_{BY} &= \int d^d x \left[ \frac{1}{12} \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} - i \tilde{\partial}_\mu \tilde{\rho}_\nu (\tilde{\partial}^\mu \rho^\nu - \tilde{\partial}^\nu \rho^\mu) + \tilde{\partial}_\mu \tilde{\sigma} \tilde{\partial}^\mu \sigma + \beta_\nu (\tilde{\partial}_\mu B^{\mu\nu} - \tilde{\partial}^\nu \varphi) \right. \\
&\quad + \epsilon (Y_\nu^* + \alpha \beta_\nu)^2 - (\tilde{\partial}_\mu Y_\nu^* - \tilde{\partial}_\nu Y_\mu^*) \tilde{\partial}^\mu Y^\nu - i \bar{\chi} \tilde{\partial}_\mu \rho^\mu - i \chi (\tilde{\partial}_\mu \bar{\rho}^\mu - \lambda_2 \bar{\chi}) \\
&\quad \left. - i \tilde{\partial}_\mu K_\nu^* (\tilde{\partial}^\mu K^\nu - \tilde{\partial}^\nu K^\mu) + \tilde{\partial}_\mu Z^* \tilde{\partial}^\mu Z \right] \\
&= \int d^d x \left[ \frac{1}{12} \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} - i \tilde{\rho}_\nu \left( \partial_\mu \partial^\mu \rho^\nu - \partial_\mu \partial^\nu \rho^\mu - m^2 \rho^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \partial \cdot \rho + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\nu n \cdot \rho \right. \right. \\
&\quad \left. \left. - \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} n^\nu n \cdot \rho \right) - \bar{\sigma} (\partial_\mu \partial^\mu - m^2) \sigma + \beta_\nu \partial_\mu B^{\mu\nu} - \frac{1}{2} m^2 \beta_\nu \frac{1}{n \cdot \partial} n_\mu B^{\mu\nu} + \epsilon (Y_\nu^* + \alpha \beta_\nu)^2 \right. \\
&\quad - \beta_\nu \partial^\nu \varphi + \frac{1}{2} \frac{m^2}{n \cdot \partial} \beta_\nu n^\nu \varphi - (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*) \partial^\mu Y^\nu + \frac{1}{2} (\partial_\mu Y_\nu^* - \partial_\nu Y_\mu^*) \frac{m^2}{n \cdot \partial} n^\mu Y^\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} (n_\mu Y_\nu^* - n_\nu Y_\mu^*) \partial^\mu Y^\nu - \frac{1}{4} \frac{m^2}{n \cdot \partial} (n_\mu Y_\nu^* - n_\nu Y_\mu^*) \frac{m^2}{n \cdot \partial} n^\mu Y^\nu - i \bar{\chi} \partial_\mu \rho^\mu + \frac{i}{2} m^2 \bar{\chi} \frac{1}{n \cdot \partial} n_\mu \rho^\mu \\
&\quad - i \lambda_2 \bar{\chi} \chi - i \bar{\rho}^\mu \partial_\mu \chi - \frac{i}{2} \frac{m^2}{n \cdot \partial} \bar{\rho}^\mu n_\mu \chi - i \partial_\mu K_\nu^* (\partial^\mu K^\nu - \partial^\nu K^\mu) + \frac{i}{2} \partial_\mu K_\nu^* \frac{m^2}{n \cdot \partial} (n^\mu K^\nu - \partial^\nu K^\mu) \\
&\quad \left. + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu K_\nu^* (\partial^\mu K^\nu - \partial^\nu K^\mu) + \frac{i}{4} \frac{m^2}{n \cdot \partial} n_\mu K_\nu^* \frac{m^2}{n \cdot \partial} n^\nu K^\mu - Z^* (\square - m^2) Z \right], \tag{50}
\end{aligned}$$

where  $K_\nu, K_\nu^*$  and  $Z, Z^*$  are the ghost fields and ghost of ghost fields, corresponding to the gaugeon fields.

The gaugeon fields and respective ghost and ghost of ghost fields change under the BRST transformations:

$$\begin{aligned}
s_b Y_\nu &= K_\nu, & s_b K_\nu &= 0, \\
s_b K_\nu^* &= i Y_\nu^*, & s_b Y_\nu^* &= 0, \\
s_b Z^* &= 0, & s_b Z &= 0.
\end{aligned} \tag{51}$$

Therefore, the gaugeon Lagrangian density (50) remains intact under the effect of combined BRST transformations (43) and (51).

The BRST charge is given by

$$\begin{aligned}
Q_B &= \int d^{d-1} x \left[ -2 \tilde{F}^{0\nu\rho} (\tilde{\partial}_0 \rho_\nu - \tilde{\partial}_\nu \rho_0) + \beta_\nu (\tilde{\partial}^0 \rho^\nu - \tilde{\partial}^\nu \rho^0) - \tilde{\partial}_\nu \sigma (\tilde{\partial}^0 \bar{\rho}^\nu - \tilde{\partial}^\nu \bar{\rho}^0) \right. \\
&\quad \left. + \bar{\chi} \tilde{\partial}^0 \sigma - \chi B^0 - K_\nu (\tilde{\partial}^0 Y^{\nu*} - \tilde{\partial}^\nu Y^{*0}) + Y_\nu^* (\tilde{\partial}^0 K^\nu - \tilde{\partial}^\nu K^0) \right], \tag{52}
\end{aligned}$$

which annihilates the physical subspace of the total Hilbert space:

$$Q_B |\text{phys}\rangle = 0. \tag{53}$$

This single subsidiary condition of Kugo-Ojima type removes both the unphysical gauge modes as well as unphysical gaugeon modes.

The gaugeon Lagrangian density (50) also admits the following quantum gauge transformations:

$$\begin{aligned}
\delta_q B_{\mu\nu} &= \tau \left( \partial_\mu Y_\nu - \partial_\nu Y_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu Y_\mu \right), \\
\delta_q \rho_\mu &= \tau K_\mu, & \delta_q \sigma &= \tau Z, \\
\delta_q Y_\nu^* &= -\tau \beta_\nu, & \delta_q K_\mu^* &= -\tau \bar{\rho}_\mu, \\
\delta_q Z^* &= -\tau \bar{\sigma}, & \delta_q \Theta &= 0, \\
\Theta &= \bar{\rho}_\mu, \beta_\mu, \bar{\sigma}, \bar{\chi}, \varphi, \chi, Y_\nu.
\end{aligned} \tag{54}$$

Under the above quantum gauge transformation, Lagrangian density (50) is form-invariant:

$$S_{BY}(\phi^A, \alpha) = S_{BY}(\hat{\phi}^A, \hat{\alpha}), \quad (55)$$

where

$$\hat{\alpha} = \alpha + \tau\alpha. \quad (56)$$

Here “ $\hat{\phantom{x}}$ ” refers to the quantum gauge transformed quantity. It is easy to see that, the quantum gauge transformations in the 2-form gauge theory (54) also commute with BRST transformations mentioned in (51). Consequently, it is confirmed that the Hilbert space, spanned from physical states, annihilated by BRST charge, is also invariant under the quantum gauge transformations:

$$\hat{Q}_B = Q_B. \quad (57)$$

Hence, the physical subspace in case of Abelian 2-form gauge theory is also invariant under quantum gauge transformation.

#### IV. CONCLUSION

In conclusion, we have investigated the gaugeon formalism in the context of VSR. It is found that the gaugeon modes together with gauge modes become massive in the VSR scenario. Our results are very general and will be valid for any (arbitrary) gauge theory. For illustration, we have first considered Maxwell theory in the VSR framework, which remains invariant under the modified gauge transformations, rather being invariant under usual gauge transformations. For a VSR-type Lorentz gauge, we have obtained a Proca type equation for Euler-Lagrange equation of motion, revealing that the gauge field gets a mass. Further, to investigate quantum gauge symmetry, we introduced the gaugeon modes to the VSR invariant Maxwell theory. Remarkably, the subsidiary condition to remove unphysical gaugeon modes satisfy the Proca type equation. This observation confirms that the gaugeon fields are also massive, where as in SR invariant theory, these gaugeon fields were massless. Therefore, the massive gaugeon fields could possibly be a candidate for dark matter, where mass appears naturally due to VSR effect. Further, we have studied the BRST symmetric gaugeon formalism and obtained only one (most general) subsidiary condition of Kugo-Ojima type. Analogously, we have derived the gaugeon formalism for Abelian 2-form gauge theory also in VSR. Here, we have found that there exist many more auxiliary fields, as it is a reducible theory and all the modes become massive with same mass. This is the reason why this mass generation is different from Higgs mechanism. It would be interesting to explore the present investigation for the higher-form gauge theories. The higher-form fields are important ingredients to certain string and super-gravity theories. From the viewpoint that the VSR symmetry is a fundamental symmetry of nature, there is no obvious reason to stop introducing additional VSR-invariant but Lorentz-violating interactions to these theories.

Other approaches for mass generation have also been studied [52, 53], involving the topological terms. However, a massive Proca theory can be imbibed with gauge invariance by introducing a scalar mode. An Abelian gauge theory describing dynamics of massive spin one bosons is studied recently, where it is found that, the theory respects Lorentz invariance, locality, causality and unitarity [54]. In Ref. [55, 56], the possibility to attain scale invariance and novel symmetries for the massive theory, through suitable coupling with scalar-tensor gravity are demonstrated. In this process, not only the photon becomes massive, but the scalar-tensor gravity also acquires non-vanishing conserved scale current. It will be fascinating to explore such a possibility from the VSR point of view.

The structure of results we obtained here by studying gaugeon formalism in VSR is not very different to that of SR case. Unlike the SR case, the novel observation is that in VSR scenario, all the fields (including gaugeon and ghost fields) acquire mass, which modifies the masses of the original dispersion relations. Present investigation might play an important role for discrete symmetry violating gauge theories. In a very recent work, following Sakoda’s treatment of Yang-Mills fields [51], the Harada-Tsutsui gauge recovery procedure [57] has been applied to the gauge non-invariant functional and Type

I and the extended Type I gaugeon formalism have been obtained [58]. We have studied the Sakoda's treatment for 1-form gauge theory in VSR by including the two gauges of the standard formalism. In this consideration, the theory describing extended gauge freedom occupies the total Fock space, which embeds the Fock spaces of both gauges. It has been found that the Harada-Tsutsui gauge recovery procedure still holds for VSR scenario. We have found that Sakoda's effective action coincides with the gaugeon action. However, Sakoda's theory cannot arbitrarily change the gauge parameter as is done by gaugeon formalism.

From a quantum field theoretic structure suitable to describe the dark matter, it has been emphasized, in Ref. [59], that VSR plays the same role for the dark matter fields as SR does for the standard model fields. From this perspective, our present investigations will also play an important role in clear understanding of the dark matter, where massive gaugeon and gauge fields as the dark matter candidates support the algebraic structure underlying VSR. Possible physical realization of these novel form of mass generation is also of interest.

- 
- [1] V. A. Kostelecky and S. Samuel, Phys. Rev. D 39 (1989) 683; V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. 97 (2006) 140401; V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. 99 (2007) 011601.
  - [2] S. M. Carroll, G. B. Field and R. Jackiw, Phys. Rev. D 41 (1990) 1231.
  - [3] S. R. Coleman and S. L. Glashow, Phys. Rev. D 59 (1999) 116008; F. W. Stecker and S. L. Glashow, Astropart. Phys. 16 (2001) 97.
  - [4] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane and T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601.
  - [5] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, JHEP 0405 (2004) 074.
  - [6] D. Colladay and V. A. Kostelecky, Phys. Rev. D 55 (1997) 6760; D. Colladay and V. A. Kostelecky, Phys. Rev. D 58 (1998) 116002; V. A. Kostelecky, Phys. Rev. D 69 (2004) 105009.
  - [7] P. Auger Collaboration, Phys. Rev. Lett. 101 (2008) 061101.
  - [8] J. Alfaro, H. Morales-Tecotl, and L. F. Urrutia, Phys. Rev. Lett. 84 (2000) 2318; Phys. Rev. D 65 (2002) 103509.
  - [9] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. 97 (2006) 021601.
  - [10] J. Vohánka and M. Faizal, Phys. Rev. D 91 (2015) 045015; Eur. Phys. J. C 75 (2015) 592; Nucl Phys B 904, 327 (2016).
  - [11] J. Vohánka, Phys. Rev. D 85 (2012) 105009.
  - [12] S. Petras, R. von Unge, J. Vohánka, JHEP 1107, (2011) 015.
  - [13] A. G. Cohen and S. L. Glashow, arXiv:hep-ph/0605036.
  - [14] A. G. Cohen and D. Z. Freedman, JHEP 07 (2007) 039; J. Vohanka, Phys. Rev. D 85 (2012) 105009.
  - [15] G.W. Gibbons, J. Gomis, and C. N. Pope, Phys. Rev. D 76 (2007) 081701.
  - [16] W. Muck, Phys. Lett. B 670 (2008) 95.
  - [17] M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. 101 (2008) 261601.
  - [18] S. Das, S. Ghosh and S. Mignemi, Phys. Lett. A 375 (2011) 3237.
  - [19] E. Alvarez and R. Vidal, Phys. Rev. D 77 (2008) 127702.
  - [20] D.V. Ahluwalia and S. P. Horvath, JHEP 11 (2010) 078.
  - [21] Z. Chang, M.-H. Li, X. Li, and S. Wang, Eur. Phys. J. C 73 (2013) 2459.
  - [22] S. Cheon, C. Lee, and S. Lee, Phys. Lett. B 679 (2009) 73.
  - [23] J. Alfaro and V. O. Rivelles, Phys. Lett. B 734 (2014) 239.
  - [24] J. Alfaro and V. O. Rivelles, Phys. Rev. D 88 (2013) 085023.
  - [25] A. Ilderton, Phys. Rev. D 94 (2016) 045019.
  - [26] R. Bufalo, Phys. Lett. B 746 (2015) 251.
  - [27] S. Upadhyay, Eur. Phys. J. C 75 (2015) 593.
  - [28] C.-Y. Lee, Phys. Rev. D 93, 045011 (2016).
  - [29] K. Yokoyama, Prog. Theor. Phys. 51 (1974) 1956.
  - [30] K. Yokoyama, Prog. Theor. Phys. 59 (1978) 1699.
  - [31] M. Hayakawa and K Yokoyama, Prog. Theor. Phys. 44 (1970) 533.
  - [32] K. Izawa, Prog. Theor. Phys. 88 (1992) 759.
  - [33] M. Koseki, M. Sato and R. Endo, Prog. Theor. Phys. 90 (1993) 1111.
  - [34] T. Kugo and I. Ojima, Prog. Theor. Phys. Supplement No. 66 (1979) 1.

- [35] T. Kugo and I. Ojima, Nucl.Phys. B 144 (1978) 234.
- [36] M. A. L. Capri, D. Dudal, D. Fiorentini, M. S. Guimaraes, I. F. Justo, A. D. Pereira, B. W. Mintz, L. F. Palhares, R. F. Sobreiro, and S.P. Sorella, Phys. Rev. D 94 (2016) 025035.
- [37] L. E. Oxman and G. C. Santos-Rosa, Phys. Rev. D 92 (2015) 125025.
- [38] M. Faizal, Found. Phys. 41 (2011) 270; Eur. Phys. J. C 71 (2011) 1603; J. Phys. A 44 (2011) 402001; Phys. Lett. B 705 (2011) 120; Phys. Rev. D 84 (2011) 106011; Mod. Phys. Lett. A 27 (2012) 1250075; JHEP 1204 (2012) 017; Europhys. Lett. 98 (2012) 31003; Mod. Phys. Lett. A 27 (2012) 1250147; Comm. Theor. Phys. 58 (2012) 704; Int. J. Theor. Phys. 52 (2013) 392; Nucl. Phys. B. 869 (2013) 598; JHEP 1301 (2013) 156; Int. J. Mod. Phys. A 28 (2013) 1350012; Mod. Phys. Lett. A 28 (2013) 1350034; Mod. Phys. Lett. A 29 (2014) 1450154; M. Faizal and D. J. Smith, Phys. Rev. D 87 (2013) 025019; P. Weinreb and M. Faizal, Phys. Lett. B 748 (2015) 102.
- [39] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, Mod. Phys. Lett. A 30 (2015) 1550108.
- [40] M. Schaden and D. Zwanziger, Phys. Rev. D 92 (2015) 025001.
- [41] M. Koseki, M. Sato and R. Endo, Nat. Sci. 14 (1996) 15.
- [42] Y. Nakawaki, Prog. Theor. Phys. 98 (1997) 5.
- [43] R. Endo and M. Koseki, Prog. Theor. Phys. 103 (2000) 3.
- [44] H. Miura and R. Endo, Prog. Theor. Phys. 117 (2007) 4.
- [45] M. Faizal, Commun. Theor. Phys. 57 (2012) 637.
- [46] M. Faizal, Mod. Phys. Lett. A 27 (2012) 1250147.
- [47] S. Upadhyay, EPL 105 (2014) 21001.
- [48] S. Upadhyay, Eur. Phys. J. C 74 (2014) 2737; Annals of Physics 344 (2014) 290.
- [49] A. C. Nayak, R. K. Verma and P. Jain, JCAP 07 (2015) 031.
- [50] M. Kalb and P. Ramond, Phys. Rev. D 9 (1974) 2273.
- [51] S. Sakoda, Prog. Theor. Phys. 117 (2007) 745.
- [52] M. Leblanc, R. Mackenzie and P. K. Panigrahi and R. Ray, Int. J. Mod. Phys. A 09 (1994) 4717.
- [53] A. Lahiri, Phys. Rev. D 55 (1997) 5045.
- [54] V. M. Vyas and V. Srinivasan, Int. J. Theor. Phys. 55 (2016) 2610.
- [55] A. Shukla, K. Abhinav and P. K. Panigrahi, Class. Quantum Grav. 33 (2016) 235008.
- [56] K. Abhinav, A. Shukla and P. K. Panigrahi, arXiv:1606.01369 [hep-th].
- [57] K. Harada and I. Tsutsui, Prog. Theor. Phys. 78 (1987) 878.
- [58] T. Saito, R. Endo and H. Miura, Prog. Theor. Exp. Phys. (2016) 023B02.
- [59] D. V. Ahluwalia and S. P. Horvath, JHEP 1011 (2010) 078.